John Fry — jf3cq

Lemma: if an integer is not even, it is odd (I will prove this by contradiction)

1) Definition: an integer x is **even** if and only if, for some integer y, x = 2y
2) Definition: an integer u is **odd** if and only if, for some integer v, u = 2v + 1
3) For contradiction purposes, let us assume that if an integer is not even, it does *not* have to be odd
4) C is defined as the set of all integers that are not even and not odd
5) From step 3, C is not empty
6) From the well-ordering principle, we can select the minimum of C and name it m
7) Since m is in C, there is no integer y for which m = 2y and no integer v for which m = 2v + 1
8) (m = 2y) is logically equivalent to (m - 2 = 2y - 2) since the same thing is being subtracted from both sides
9) Since there is no y for which m = 2y, there is no y for which m - 2 = 2y - 2
10) There is no y for which m - 2 = 2(y - 1)
11) Since y can take any integer value, there is also no y for which m - 2 = 2y. If there were a y for which m - 2 = 2y, then there would also be a y' for which m - 2 = 2(y' - 1), which would contradict step 10. Specifically, y' would equal y+1.
12) Since there is no y for which m - 2 = 2y, **m - 2 is not even**.
13) (m = 2v + 1) is logically equivalent to (m - 2 = 2v - 2 + 1) since the same thing is being subtracted from both sides
14) Since there is no v for which m = 2v + 1, there is no v for which m - 2 = 2v - 2 + 1
15) There is no v for which m - 2 = 2(v - 1) + 1
16) Since v can take any integer value, there is also no v for which m - 2 = 2v + 1. If there were a v for which m - 2 = 2v + 1, then there would also be a v' for which m - 2 = 2(v' - 1) + 1, which would contradict step 15. Specifically, v' would equal v + 1.
17) Since there is no v for which m - 2 = 2v + 1, **m - 2 is not odd**.
18) From steps 12 and 17, m - 2 is neither even nor odd.
19) Since m - 2 is neither even nor odd, m - 2 is in C.
20) Since m - 2 is in C, m is not the minimum element of C.
21) This is a contradiction! Assuming our lemma to be false led to this contradiction, so our lemma is proved.

QED