Class 18: Spooky Infinities

Schedule

Problem Set 7 is due **Friday (27 Oct) at 6:29pm**.

Exam 2 is two weeks from today (November 9, in class). We will post more information about Exam 2 soon.

Countable and Uncountable Sets

Definition. A set S is *countably infinite* if and only if there exists a bijection between S and $\mathbb{N}$.

Definition. A set S is *uncountable*, if there exists no bijection between S and $\mathbb{N}$.

The **power set** of A (pow(A))is the set of all subsets of A:

$$B \in \text{pow}(A) \iff B \subseteq A.$$

For all **finite** sets S, $|pow(S)| = 2^{|S|}$.

For **all** sets S, $|pow(S)| > |S|$.

Prove $pow(\mathbb{N})$ is uncountable.

bitstrings $= \forall n \in \mathbb{N}.\{0,1\}^n$.

Ordinal and Cardinal Numbers

ω is the *smallest infinite ordinal.* The first ordinal after $0, 1, 2, \cdots$.
What is the difference between an *ordinal* and *cardinal* number?

What should 2ω mean?

Is InfiniteBitStrings $= \{0,1\}^{\omega}$ countable?

Prove the number of real numbers in the interval $[0, 1]$ is uncountable.

