# Class 19: Reviewing Infinities

**Schedule**

**Problem Set 8** is now due on **Friday, Nov 3**.

See PDF Version for Notes.

## Comparing sets Recap:

**Definition.** Sets $A$ and $B$ have the same cardinality, denoted by $|A|=|B|$ if there is a bijection between $A$ and $B$.

**Definition.** Cardinality of $A$ is at least as big as cardinality of $B$, denoted by $|A|\geq|B|$ if and only if *either* of the following is true (they are equivalent).

1. There is a surjective function from $A$ to $B$.
2. There is a total injective function from $B$ to $A$.

## Infinite Sets Recap

**Definition.** A set $C$ is *infinite* if and only if *either* of the following happens (they are all equivalnet).

1. Dedekind-infinite: There is a bijection between $C$ and a strict subset $B$ of $C$.
2. There is *no* bijection between $C$ and any $\mathbb{N}_k$ for any natural number $k \in \mathbb{N}$.
3. There exists a surjective function from $C$ to $\mathbb{N}$.
4. There exists a total injective function from $\mathbb{N}$ to $C$.

**Definition.** A set $C$ is *countable* if and only if there exists a surjective function from $\mathbb{N}$ to $C$. (That is, $\leq 1$ arrow out from $\mathbb{N}$, $\geq 1$ arrow in to $C$.)

**Definition.** A set $C$ is *countably infinite* if and only if there exists a bijection between $C$ and $\mathbb{N}$.

## Cantor's Theorem

For **all** sets, $S$, $|pow(S)|>|S|$.

What does this mean for $|\mathbb{N}|$?

Show there is a bijection between $[0, 1]$ and $pow(\mathbb{N})$.

What is the cardinality of all the real numbers? Show a bijection between $[0, 1]$ and all real numbers. Hint, first show a bijection between $(0, 1)$ and real numbers, and then a bijection between $[0, 1]$ and $(0, 1)$.

  