Class 5: CNF, Computing, Quantifiers

Schedule

Problem Set 2 is due **Friday at 6:29pm**.

Notes and Questions

Definition: satisfiable. A logical formula is *satisfiable* if there is *some* way to make it **true**. That is, there is at least one assignment of truth value to its variables that makes the forumla true.

Definition: conjunctive normal form (CNF). A logical formula that is written as a conjunction of *clauses*, where each clause is a disjunction of *literals*, and each literal is either a variable or a negation of a variable, is in *conjunctive normal form*. If each clause has excatly three literals, it is called *three conjunctive normal form* (3CNF).

$$(a_1 \vee a_2 \vee \neg a_3) \wedge (a_1 \vee \neg a_2 \vee a_3) \wedge (\neg a_1 \vee a_2 \vee \neg a_3) \wedge (\neg a_1 \vee a_2 \vee a_3)$$

Show that every logical formula can be written in conjunctive normal form. Also, show that if we only care about *satisfiability* we can always write it in 3CNF form. Namely, for every CNF formula F, we can write a 3CNF formula G such that F is satisfiable if and only if G is satisfiable.

What is the maximum number of (different) clauses in a 3CNF formula involving 5 variables?

What is the maximum number of (different) clauses in a *satisfiable* 3CNF formula involving 5 variables?

What is the maximum number of (different) clauses in a *valid* 3CNF formula involving 5 variables?

Logical Quantifiers

$\forall x \in S.P(x)$ means P holds for *every* element of S.
$\exists x \in S.P(x)$ means P holds for *at least one* element of S.

Define *valid* and *satisfiable* using logical quantifiers:

$\forall x \in S.P(x)$ is equivalent to $\neg(\exists x \in S. \qquad\qquad)$

Notation: pow(S) denotes the *powerset* of S. The powerset of a set is the set of all possible subsets of that S. So, pow$(\mathbb{N})$ denotes all subsets of the natural numbers.

Notation: $A - B$ denotes the *difference* between two sets. It is the elements of A, with every element of B removed.

Notation: $\varnothing$ is the *empty set*. It is the set with no elements: $\{\}$.

$$\_\_\_\_ S \in \text{pow}(\mathbb{N}) - \{\varnothing\}. \_\_\_\_ m \in S. \_\_\_\_ x \in S - \{m\}. m < x$$

