Jake Smith (jts5np)
September 6th, 2017

Objective: Find the shortest formula that is equivalent to XOR(a, b) using just NAND operations. Shortest means the minimum number of NAND operations. A convincing answer would include a proof that no shorter formula exists.

Solution: The below program uses recursion to brute force all possible combinations of NAND gates that would result in a valid expression that represents the XOR function. The results show that the minimum number of NAND gates needed is 5. The results that are printed out can be directly copied and pasted into Wolfram Alpha if you would like to verify the results with a 3rd party; however, solutions with '1's, for some reason, are not read by Wolfram Alpha correctly (there are plenty of solutions that do not use '1's though!). Since there are a (relatively) small number of potential solutions to minimize the number of NANDs used, a proof by cases is possible here.

In order to try all combinations to make the shortest NAND expression for XOR, I recognized that there were 5 possible inputs to each argument of the NAND function for 2 variables: a, b, T, F, NAND(XYZ, XYZ) (T and F are represented below by 1 and 0 respectively). The 0 is not needed since it will provide the same result as the variable paired with it, so we can narrow the number of possible options down to 4. From there, the script starts with 1 "top-level" NAND and tries each possibility (a, b, T, NAND(XYZ, XYZ) for every argument. By building each of the cases recursively, it allows us to easily generate all the possibilities at each stage. The cases are then evaluated at each step to check if they are logically equivalent to XOR. In order to minimize the number of NANDs used, the script does not add anymore NANDs after 5 have been used in the expression.

Note: The below code is for Python 2.7. To make it Python3 compatible, I think only the print statement in the nand function would need to be changed to be of the form print(stuff). Note the inclusion of the parentheses, as print is a function in Python3.

Program:

```
#NAND BruteForce
#Author: Jake Smith (jts5np)
#Date: 9-5-2017
#Last Date: 9-10-2017
#Class: Discrete Math

def nand(exp):
    code1 = 'a = 1; b = 1; ' + exp
    code1 = code1.replace('print','f1 =')
    code2 = 'a = 1; b = 0; ' + exp
    code2 = code2.replace('print','t1 =')
    code3 = 'a = 0; b = 1; ' + exp
    code3 = code3.replace('print','t2 =')
    code4 = 'a = 0; b = 0; ' + exp
    code4 = code4.replace('print','f2 =')
    exec(code1)
    exec(code2)
```

```
    exec(code3)
    exec(code4)
    if (t1 and t2) and not (f1 or f2):
        exp = exp.replace('not (','NAND(')
        exp = exp.replace(' and ',', ')
        exp = exp.replace('print','')
        print 'NANDs used: ' + str(exp.count('NAND')) + ' Expression: ' + exp

def brute_force(string):
    if string.count('{}') == 0:
        nand(string)
        return
    else:
        s1 = string.replace('{}','a',1)
        s2 = string.replace('{}','b',1)
        s3 = string.replace('{}','1',1)
        brute_force(s1)
        brute_force(s2)
        brute_force(s3)
        if string.count('not') < 5:
            s4 = string.replace('{}','not ({} and {})',1)
            brute_force(s4)

init = 'print not ({} and {})'

brute_force(init)
```

Output: (They can be easily copied into Wolfram Alpha for verification except if they have a 1 in them)

NANDs used: 5 Expression: NAND(NAND(a, NAND(a, b)), NAND(b, NAND(a, a)))
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NANDs used: 5 Expression: NAND(NAND(NAND(1, b), a), NAND(b, NAND(b, a)))
NANDs used: 5 Expression: NAND(NAND(NAND(1, b), a), NAND(b, NAND(1, a)))
NANDs used: 5 Expression: NAND(NAND(NAND(1, b), a), NAND(NAND(a, a), b))
NANDs used: 5 Expression: NAND(NAND(NAND(1, b), a), NAND(NAND(a, b), b))
NANDs used: 5 Expression: NAND(NAND(NAND(1, b), a), NAND(NAND(a, 1), b))
NANDs used: 5 Expression: NAND(NAND(NAND(1, b), a), NAND(NAND(b, a), b))
NANDs used: 5 Expression: NAND(NAND(NAND(1, b), a), NAND(NAND(1, a), b))