Problem Set 9

Deliverable: Submit your responses as a single, readable PDF file on the collab site before **6:29pm** on **Wednesday, 23 November**.

Collaboration Policy - Read Carefully

The collaboration policy is identical to that for PS7: you should work in groups of *one* to *four* students of your choice with no restrictions, and follow the rest of the collaboration policy from PS3.

Preparation

This problem set focuses on infinite sets — Chapter 8 of the MCS book, and Class 18, Class 19, Class 21 and Class 22.

Directions

Solve all 8 problems. Your answers should be clear, consise, and convincing.

Countable Sets

For each set defined below, prove that the set described is *countable*.

1. *Evens* $= \{2n \mid n \in \mathbb{N}\}$
2. $\mathbb{N} \cup \{\pi, \tau\}$ (where π is the ratio of a circle's circumference to its diameter and τ is the ratio of a circle's circumference to its radius)
3. The set of all finite state machines, $M = (S, G, q_0)$ where S is a finite set, and $G \subseteq S \times S$ and $q_0 \in S$ are otherwise unrestricted.

Possibly Countable Sets

For each set defined below, determine if the set is *countable* or *uncountable* and support your answer with a convincing proof.

4. The set of all *stree* objects, defined by:
   - Base object: **null** is an *stree*.
   - Constructor: for any *stree* objects q_1, q_2, combine(q_1, q_2) is an *stree*.

5. $\mathbb{R} - \mathbb{Q}$.

6. ($\star$) The set of all *infinite* state machines, $M = (S, G, q_0)$ where $S = \mathbb{N}$, and $G \subseteq S \times S$ and $q_0 \in S$ are otherwise unrestricted.

Properties of Infinite Sets

7. (MCS Problem 8.11, with typesetting problem fixed)

   (a) Prove that if a nonempty set, C, is countable, then there is a *total* surjective function,

   $$f : \mathbb{N} \to C.$$

   (b) Conversely, suppose that $\mathbb{N}$ surj D, that is, there is a surjective function that is *not necessarily total*, $f : \mathbb{N} \to D$. Prove that D is countable.

8. MCS Problem 8.17.

